

RBO Protocol: Broadcasting Huge Databases for Tiny Receivers

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Abstract—We propose a protocol (called RBO) for broadcasting long streams of single-packet messages over radio channel for tiny, battery powered, receivers. The messages are labeled by the keys from some linearly ordered set. The sender repeatedly broadcasts a sequence of many (possibly millions) of messages, while each receiver is interested in reception of a message with a specified key within this sequence. The transmission is arranged so that the receiver can wake up in arbitrary moment and find the nearest transmission of its searched message. Even if it does not know the position of the message in the sequence, it needs only to receive a small number of (the headers of) other messages to locate it properly. Thus it can save energy by keeping the radio switched off most of the time. We show that bit-reversal permutation has “recursive bisection properties” and, as a consequence, RBO can be implemented very efficiently with only constant number of $\lceil \log_2 n \rceil$ -bit variables, where n is the total number of messages in the sequence. The total number of the required receptions is at most $2\lceil \log_2 n \rceil + 2$ in the model with perfect synchronization. The basic procedure of RBO (computation of the time slot for the next required reception) requires only $O(\log^3 n)$ bit-wise operations. We propose implementation mechanisms for realistic model (with imperfect synchronization), for operating systems (such as e.g. TinyOS).

Index Terms—Radio network, broadcast scheduling, energy efficiency.

I. INTRODUCTION

Recursive Bisection Ordering (RBO) Protocol is a protocol, based on a very simple ranking algorithm [1], for a powerful sender and energetically tiny receivers. The sender repeatedly broadcasts a sequence of messages. Each message is labeled by a key. The time intervals between subsequent starts of message transmissions in the sequence are equal. We call them *time slots*. At arbitrary time moment the user of RBO (i.e. some application running on the receiver device) may ask the RBO module to receive a message with some specified key. Since then, the task of the RBO module is to receive the nearest transmission of the message labeled with this key and deliver this message to the user. The simplest strategy would be keeping the radio switched on and listen to all messages until the searched one is received. However, radio consumes a lot of energy while it is switched on and the receiver device has a limited energy source (i.e. battery). If the whole sequence contains millions of messages, then we may need to wait many hours until the searched message is transmitted. Therefore we need a strategy that minimizes the total radio working time and does receive the nearest transmission of the searched message.

Finding broadcast scheduling that optimizes energy consumption in the battery powered receivers becomes one of the main problems in diverse modern applications. An example is a very recent algorithm of finding optimal scheduling of broadcast bursts for mobile TV channels [2].

Other example are wireless networks of battery powered sensors. Nodes of such network consist of possibly simple processor, a very limited memory, specialized sensing or measurement tools, and radio receiver and transmitter. Usually, the task of such network is reporting the measurements or detected events to the base station. The radio receiver can be used for forwarding packets from the other more distant sensors, since the range of the sensor’s transmitter is in many cases shorter than the distance to the base station (to save the energy). The other application of the radio receiver can be receiving control messages from the base station. However, keeping the radio receiver switched on all the time would consume too much energy. Techniques for sensor networks such as Low Power Listening (LPL, [3]), where the receiver samples for short periods radio channel and continues listening if it detects any transmission, while the sender transmits a sequence of few copies of the message to ensure one successful reception, are appropriate for an extensively used channel. On the other hand, RBO is appropriate for a channel with continuous stream of messages, where each receiver wants to receive only few of them. Also the sleeping intervals for LPL are constant (and so are the energy savings), while RBO flexibly adapts the sleeping intervals. They can be very long for very long sequences of messages.

RBO can be used for transmission of public large databases that can be accessed by battery powered devices such as palm-tops. However, the efficiency and simplicity of its implementation makes it also useful for very weak devices such as sensors. For example, it enables sending control commands to a great multitude of sensors over a single radio channel. Each receiver can use RBO to filter its own messages without any prior knowledge about the transmission schedule. In such system, we can add/remove receivers without affecting the behavior of the other receivers. Thus, we have a simple and flexible mechanism for time-division multiplexing of messages on a single radio channel. Note that in future we may face the problem of broadcasting of a very large amounts of information to multitude of energy constrained devices scattered in our solar system. The only transmission medium would be limited number of radio channels.

Another application of the RBO can be centralized channel access control for upload transmissions (e.g. for overcrowded channel): The base station broadcasts only the headers, while the rest of the time slot can be used for transmission by

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the (unique) owner of the key from the header. It can also be considered for broadcasting interrogation signals for reporting selective readings from sensors or battery powered (gas/water) meter devices. This could be generalized to the idea of distributed algorithms performed by sensors (such as e.g. routing towards the base station) assisted by a powerful base station broadcasting control/synchronization commands organizing the distributed computation.

Transmitting large database for battery powered receivers has been considered by Imielinski, Viswanathan and Badrinath in [4], [5], and [6]. They proposed several techniques based on hashing and inclusion of indexing informations in data stream that let the receiver energetically efficient searching for data.

Specific variants of the problem and efficiency measures have also been considered: Broadcast scheduling minimizing latency in the presence of errors has been considered in [7]. In [8] data-caching for energy saving has been proposed. Energy efficient indexing for several types of data formats has been proposed in [9], [10], [11].

We believe that, in many applications, RBO can be a more implementable and robust solution. In RBO, each message, consisting of the header and data field, is of the same type, and occasional losses of messages do not cause severe consequences.

The RBO protocol is based on a simple ranking algorithm for single hop radio network proposed in [1]. The sender sorts the messages by their keys and then permutes them by a special permutation (called *recursive bisection ordering* or *rbo*). Such sequence is periodically broadcast. The receivers' RBO protocol keeps an interval $[minR, maxR]$ of possible ranks of the searched key in the transmitted sequence. Initially $[minR, maxR] = [0, n - 1]$, where n is the length of the sequence. RBO tries to receive *only* the messages with the keys ranked in $[minR, maxR]$. Each such message is either the searched one or it can be used for further updating (shrinking) of the interval. It has been shown in [1] that no more than $4 \lg_2 n$ messages are required to locate the rank of the key in the sequence if the sequence is retransmitted in rounds, even when the search is started in arbitrary time slot.

In this paper we show that a simple bit-reversal permutation (famous for its application in FFT [12]) has the essential "recursive bisection" properties of the (recursively defined) *rbo*. This enables very efficient and simple implementation of the functions needed by the RBO protocol. Hence, RBO can be implemented on very weak devices with tiny memory resources (such as e.g. sensors).

In section II we show the properties of bit-reversal permutation that are relevant for our protocol. We also present the outline of the underlying algorithm.

In section III we show precise upper bound on the number of necessary receptions required to reach the searched message. The bound is $2 \lceil \lg_2 n \rceil + 2$. Due to the simpler permutation and more detailed proof, this bound is lower than the one in [1]. We show an example, when $2 \lceil \lg_2 n \rceil - 1$ receptions are required. We also include experimental results of the simulations, in the case when the communication is unreliable.

In section IV we propose simple and efficient algorithm for computing the time-slot of the next message that should be

received by the receiver. The algorithm enables computations for very long sequences of messages (possibly many millions or more) even on very weak processors. It requires $O(\log^3 n)$ bit-wise operations and a constant number of $\lceil \log_2 n \rceil$ -bit variables.

In section V we discuss the implementation of the protocol on real devices. A prototype of the protocol with a simple demonstration application has been implemented in Java language and is available at [13]. This implementation is designed to be easily transformable to TinyOS ([14], [15]): the required modules of TinyOS, hardware components and radio channel has been modelled by appropriate objects. RBO protocol offers split-phase interface to the user. The user issues a command to find a message with given key and, after some time is signalled the call-back with the results of the search. In the meantime RBO switches the radio receiver on and off: on the one hand – to save energy, on the other hand – to ensure the reception of all the messages required for the search. Also the basic protocol functions have been implemented with no recursion and optimized up to the bitwise operations.

II. PRELIMINARIES AND RELEVANT PROPERTIES OF BIT-REVERSAL

There is a single *broadcaster* and arbitrary number of receivers. The broadcaster has a set of n *messages* to be broadcast labeled by *keys* from some linearly ordered universe. The keys do not have to be distinct. The broadcaster sorts the messages by the values of their keys. By a *rank* we mean a position index of an item in this sorted sequence. (The positions are numbered from 0 to $n - 1$.) Then the broadcaster broadcasts in a round-robin fashion the sorted sequence of messages permuted by a fixed permutation π , i.e.: the message with rank x is broadcast in the time slots that are congruent modulo n to $\pi(x)$. On the other hand, each receiver can at arbitrary time slot start the Algorithm 1 described below (technical re-formulation of ranking proposed in [1]) to receive the message with a specified key.

We assume that the length of the transmitted sequence is $n = 2^k$, for some positive integer k . (If the actual number of messages is not a power of two, then we can duplicate some of them to obtain a sequence of length 2^k .)

For $k \geq 0$ and $x \in \{0, \dots, 2^k - 1\}$ we define:

$$revBits_k(x) = \sum_{i=0}^{k-1} 2^i \cdot x_{k-1-i},$$

where $x_i = \lfloor x/2^i \rfloor \bmod 2$. Note that if $(x_{k-1}, \dots, x_0)_2$ is a binary representation of x , then $(x_0, \dots, x_{k-1})_2$ is a binary representation of $revBits_k(x)$. We call $revBits_k$ a *k-bit-reversal permutation*.

We argue, that bit-reversal is a good choice, for the permutation π mentioned above, for the following reasons:

- The low energetic costs of the radio operation of the receiver (see Section III).
- The simplicity and efficiency of the implementation of the function *nextSlotIn* (see Section IV and [13]).

- Also the results of simulations (see Figure 2) show the robustness of the algorithm to random losses of messages, e.g. caused by external interferences.

A natural efficient solution to the problem of finding a key in the sorted sequence is application of the *binary searching*. We can define an (almost) balanced binary search tree on 2^k nodes. As the first approach we define a permutation bs_k (see the upper left graph on Figure 1). For $k \geq 0$, let bs_k (*binary search ordering*) be a permutation of $\{0, \dots, 2^k - 1\}$ defined as follows:

- $bs_0(x) = 0$, and,
- $bs_{k+1}(x) = (1 - (x \bmod 2)) \cdot bs_k(\lfloor x/2 \rfloor) + (x \bmod 2) \cdot (2^k + \lfloor x/2 \rfloor)$.

The domain of the permutation corresponds to *ranks*, while its range corresponds to *time slots*. In the definition of bs_{k+1} , for each even rank x , only the component: “ $(1 - (x \bmod 2)) \cdot bs_k(\lfloor x/2 \rfloor)$ ” can be non-zero, and, for each odd rank x , only the component: “ $(x \bmod 2) \cdot (2^k + \lfloor x/2 \rfloor)$ ” can be non-zero. Thus, all the even ranks, permuted by bs_{k-1} (ignoring the least significant – parity – bit), are placed before the odd ones – the *leaves* of *binary search tree*. The upper-left graph on Figure 1 is the graph of bs_k for $k = 5$. The axis of the range (the vertical axis) is directed *downwards*. The line segments form the binary search tree. A node (x, y) on the graph is on the level $\lceil \lg_2(y + 1) \rceil$ of the binary search tree. If the sender transmits a sorted sequence of length 2^k permuted by bs_k and the receiver starts listening in time slot zero, then it needs to receive no more than k keys to locate its searched key. However, if the receiver starts at arbitrary time, then it may be forced to receive many messages. (Consider the case, when the receiver starts in time slot 2^{k-1} and the searched key is greater than all the keys of the sequence.) In *binary search* it is essential, that all the nodes from one level precede all the nodes from the next level. However, the ordering of the nodes within each level may be arbitrary. Note that *revBits* satisfies the following recurrences:

- $revBits_0(x) = 0$, for $x = 0$, and,
- $revBits_{k+1}(x) = revBits_k(\lfloor x/2 \rfloor) + (x \bmod 2) \cdot 2^k$, for $0 \leq x \leq 2^{k+1} - 1$.

In the definition of $revBits_{k+1}$, both the set of odd ranks (mapped to the time slots $\{2^k, \dots, 2^{k+1} - 1\}$ – the leaves) and the set of the even ranks (mapped to the time slots $\{0, \dots, 2^k - 1\}$ – the part of the tree above the leaves) are both permuted recursively by $revBits_k$ (ignoring the parity bit) within their ranges of time slots. Hence, by the recursion, each level l (of size $\lceil 2^{l-1} \rceil$) is *recursively permuted* by $revBits_{l-1}$. The nodes within level l of the binary search tree form a binary search tree and the same holds for the sub-levels of the level. The binary search tree for $revBits_5$ and the trees for its levels (the first level of recursion) are shown on the graphs on Figure 1.

The *binary search tree* of $revBits_k$ has $k+1$ levels: $0, \dots, k$. By the *level* of the *time slot* t we mean $\lceil \lg_2(t + 1) \rceil$, and by the *level* of the *rank* x we mean $\lceil \lg_2(revBits_k(x) + 1) \rceil$. For each rank x on level l , we have $0 \leq x < 2^k$ and $x = 2^{k-l} + i_x \cdot 2^{k-l+1}$, for some integer i_x called *coordinate of x within level l* . Note that $i_x = \lfloor x/2^{k-l+1} \rfloor$.

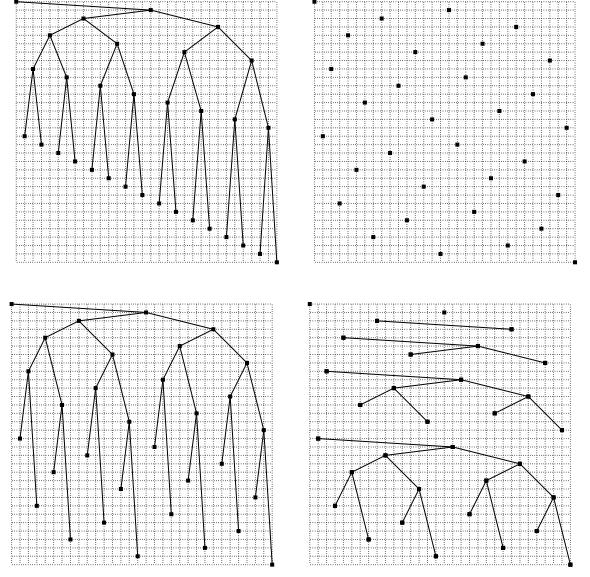


Fig. 1. The graphs of permutations: bs_5 with embedded tree (upper left), $revBits_5$ (upper right), $revBits_5$ with embedded tree (lower left), and with the trees on the first recursion level (lower right). On the graphs, the axis of domain (corresponding to *ranks*) is directed rightwards, while the axis of the range (corresponding to *time*) is directed downwards.

We use notation (a_1, a_2, \dots, a_m) to denote a sequence of the elements a_1, a_2, \dots, a_m . Thus, $()$ denotes an empty sequence. For sequences α_1 and α_2 , let $\alpha_1 \cdot \alpha_2$ denote the concatenation of α_1 and α_2 . Let $|\alpha|$ denote the length of the sequence α . For a decreasing sequence α of numbers from $\{0, \dots, k\}$, we define the set Y_α^k as follows:

- 1) $Y_{()}^k = \{0, 1, \dots, 2^k - 1\}$.
- 2) for $0 \leq l \leq \lg_2 |Y_\alpha^k|$, $Y_{\alpha \cdot (l)}^k = \{y \mid \lceil \lg_2(y - \min Y_\alpha^k + 1) \rceil = l\}$.

We use Y_α^k to denote the subsets of time slots. $Y_{()}^k$ is the set of all the time-slots and $Y_{\alpha \cdot (l)}^k$ is the set of time slots on the l th level of the binary search tree Y_α^k . The following properties are simple consequence of the definition:

- Lemma 2.1:**
- 1) $|Y_{\alpha \cdot (0)}^k| = 1$ and, for $0 < l \leq \lg_2 |Y_\alpha^k|$, $|Y_{\alpha \cdot (l)}^k| = 2^{l-1}$.
 - 2) Y_α^k is a disjoint union of the sets $Y_{\alpha \cdot (l)}^k$, where $0 \leq l \leq \lg_2 |Y_\alpha^k|$.
 - 3) $y \in Y_{\alpha \cdot (l)}^k$ if and only if $\min Y_\alpha^k + \lfloor 2^{l-1} \rfloor \leq y < \min Y_\alpha^k + 2^l$.
 - 4) $\min Y_{(l_0, l_1, \dots, l_r)}^k = \sum_{i=0}^r \lfloor 2^{l_i-1} \rfloor$.

Let $X_\alpha^k = revBits_k(Y_\alpha^k)$ – the set of the *ranks* of the time slots Y_α^k .

Let us define $step_\alpha^k$ as follows:

- if $\alpha = ()$ then $step_\alpha^k = 1$, else
- if $\alpha = \alpha' \cdot (l)$ then $step_\alpha^k = 2^{k-l+1}$.

Lemma 2.2: For each X_α^k , we have:

- 1) $x \in X_\alpha^k$ if and only if $x = (\min X_\alpha^k + i \cdot step_\alpha^k) \bmod 2^k$, for some integer i .
- 2) $step_\alpha^k \geq \min X_\alpha^k + 1$.
- 3) $\max X_\alpha^k + step_\alpha^k \geq 2^k$.

Proof: If $\alpha = ()$, then $X_\alpha^k = \{0, 1, \dots, 2^k - 1\}$ and the lemma follows. Otherwise, $\alpha = \alpha' \cdot (l)$, for some α'

and l . If $l \in \{0, 1\}$, then $|X_\alpha^k| = 1$, $step_\alpha \in \{2^{k+1}, 2^k\}$ and the lemma follows. Otherwise, $y \in Y_\alpha^k$ if and only if $\min Y_\alpha^k + 2^{l-1} \leq y < \min Y_\alpha^k + 2^l$. Note that α is decreasing and, by Lemma 2.1(4), $\min Y_\alpha^k = \min Y_{\alpha'}^k + 2^{l-1}$ is divisible by 2^{l-1} . In other words: Y_α^k (respectively, in X_α^k) is the set of all the numbers that have the $k-l+1$ most significant (respectively, least significant) bits identical to $\min Y_\alpha^k$ (respectively, $revBits_k(\min Y_\alpha^k)$). Hence, $X_\alpha^k = revBits_k(Y_\alpha^k) = \{x \mid 0 \leq x < 2^k \wedge x \bmod 2^{k-l+1} = revBits_k(\min Y_\alpha^k)\}$. Since $step_\alpha = 2^{k-l+1}$, we have

- $step_\alpha > revBits_k(\min Y_\alpha^k) = \min X_\alpha^k$ and
- $x \in X_\alpha^k$ if and only if $x = (\min X_\alpha^k + i \cdot step_\alpha) \bmod 2^k$, for some integer i .

Thus the lemma follows. \blacksquare

Notice that, for $l > 0$, $x \in X_{\alpha \cdot (l)}^k$ if and only if $x - step_{\alpha \cdot (l)}^k / 2 \in \bigcup_{i=0}^{l-1} X_{\alpha \cdot (i)}^k$. (The ranks from the level $X_{\alpha \cdot (l)}^k$ are *equidistantly interleaved* with the ranks from all previous levels $\bigcup_{i=0}^{l-1} X_{\alpha \cdot (i)}^k$.) Thus:

Lemma 2.3: For $l \geq 0$, $x \in \bigcup_{i=0}^l X_{\alpha \cdot (i)}^k$ if and only if $x = (\min \bigcup_{i=0}^l X_{\alpha \cdot (i)}^k + j \cdot 2^{k-l}) \bmod 2^k$, for some integer j .

For integer x and set of ranks X , let $\delta(x, X) = \min(\{\infty\} \cup \{d > 0 \mid x + d \in X\})$, and, for non-empty X , let $minStep(X) = \min(\{\delta(x, X) \mid x \in X\})$. If X is a singleton, then $minStep(X) = \infty$. From Lemmas 2.2 and 2.3, we have:

- Lemma 2.4:** 1) $minStep(X_\alpha) \geq step_\alpha$, and
2) $minStep(\bigcup_{i=0}^l X_{\alpha \cdot (i)}^k) \geq 2^{k-l}$.

We also state the following simple fact:

Lemma 2.5: If $2 \cdot minStep(X) > r_2 - r_1$, then $||[r_1, r_2] \cap X| \leq 2$.

A. Outline of the Protocol

The most important function used by the RBO protocol is $nextSlotIn_k$ defined, for $0 \leq t < 2^k$, $0 \leq r_1 \leq r_2 < 2^k$, as follows:

$$nextSlotIn_k(t, r_1, r_2) = (t + \tau_k(t, r_1, r_2)) \bmod 2^k,$$

where $\tau_k(t, r_1, r_2) = \min\{d > 0 : r_1 \leq revBits_k((t + d) \bmod 2^k) \leq r_2\}$. (I.e. the number of the next slot after t with rank in $[r_1, r_2]$.)

The sender simply sorts the sequence of messages by the keys and permutes it by the permutation $revBits_k$. Then it repeatedly broadcasts such sequence. The receiver contains variables $minr$ (initiated to 0), $maxr$ (initiated to $n-1$) and the searched key κ . The underlying algorithm for the receiver is outlined in Algorithm 1. Thus, the interval $[minr, maxr]$ of possible ranks of the searched key κ shrinks until it becomes empty or the searched key is found. The sleeping periods between subsequent receptions rapidly increase as the length of the interval decreases.

III. BOUNDS ON TIME AND ENERGY

Theorem 3.1: Let $n = 2^k$, for some positive integer k . Let $\kappa_0, \dots, \kappa_{n-1}$ be a sorted sequence of keys. Let κ be arbitrary searched key, let t_0 be arbitrary time slot, $0 \leq t_0 < n$,

repeat

receive message m ;

(* m contains a key $m.\kappa$ and $m.rank$ – the rank of $m.\kappa$ *)

if $m.\kappa = \kappa$ **then**

└ report the found message m and stop;

if $m.\kappa < \kappa \wedge minr \leq m.rank$ **then**

└ $minr \leftarrow m.rank + 1$;

if $m.\kappa > \kappa \wedge maxr \geq m.rank$ **then**

└ $maxr \leftarrow m.rank - 1$;

if $minr \leq maxr$ **then**

└ let $t = revBits_k(m.rank)$;

sleep (and skip all transmissions) until the

time slot $nextSlotIn_k(t, minr, maxr)$;

until $minr > maxr$;

report the absence of κ ;

Algorithm 1: Outline of the receiver's algorithm.

and, let $minr_0 = 0$ and $maxr_0 = n-1$. For $i \geq 0$, let $t_{i+1} = nextSlotIn(t_i, minr_i, maxr_i)$, and,

- if $\kappa < \kappa_{revBits(t_{i+1})}$ then $minr_{i+1} = minr_i$ and $maxr_{i+1} = revBits(t_{i+1}) - 1$, else
- if $\kappa > \kappa_{revBits(t_{i+1})}$ then $minr_{i+1} = revBits(t_{i+1}) + 1$ and $maxr_{i+1} = maxr_i$, else
- $minr_{i+1} = minr_i$ and $maxr_{i+1} = maxr_i$.

Let $e = \min\{i > 0 \mid minr_i \geq maxr_i \vee \kappa_{revBits(t_i)} = \kappa\}$. We have:

- 1) $e \leq 2 \lg_2 n + 2$, and
- 2) t_e is at most n time slots after t_0 .

Proof:

Note that $t_1 = (t_0 + 1) \bmod n$, and t_1, t_2, \dots, t_e are the reception time slots required by the search for κ started just before t_1 . If $\kappa \in \{\kappa_0, \dots, \kappa_{n-1}\}$, then the sequence $(t_1, t_2, \dots, t_{e-1}, t_e)$ is a prefix of the sequence of time slots used for searching for some $\kappa' \notin \{\kappa_0, \dots, \kappa_{n-1}\}$ with the same rank as κ . Therefore we consider only the case: $\kappa \notin \{\kappa_0, \dots, \kappa_{n-1}\}$.

Note that $\{\kappa_{t_1}, \kappa_{(t_1+1) \bmod n}, \dots, \kappa_{(t_1+n-1) \bmod n}\}$ contains all the keys $\kappa_0, \dots, \kappa_{n-1}$. Hence, the bound on time (part 2) is valid.

Now consider the part 1 (the bound on energy). Let U denote the set of the (used) time slots $\{t_1, t_2, \dots, t_{e-1}, t_e\}$.

Let T_i be the set of all the time slots since t_1 until $(t_{i+1} - 1) \bmod n$: $T_0 = \emptyset$ and, for $1 \leq i < e$, $T_i = \{(t_1 + d) \bmod n \mid 0 \leq d < d_i\}$, where $d_i = \min\{x \geq 0 \mid t_{i+1} = (t_1 + x) \bmod n\}$. Let $R_i = revBits_k(T_i)$ be the ranks of the time slots T_i . Lemma 3.1 follows from the definition of $nextSlotIn_k$ and $minr_i$ and $maxr_i$:

Lemma 3.1: The values $minr_i - 1$ and $maxr_i + 1$ are the most precise bounds on the rank of κ from the subset $R_i \cup \{-1, n\}$:

- 1) $minr_i - 1 = \max(\{-1\} \cup \{x \mid \kappa_x < \kappa \wedge x \in R_i\})$, and
- 2) $maxr_i + 1 = \min(\{n\} \cup \{x \mid \kappa_x > \kappa \wedge x \in R_i\})$, and
- 3) (since $\kappa \notin \{\kappa_0, \dots, \kappa_n\}$) $maxr_i + 1 = minr_i - 1 + \delta(minr_i - 1, \{n\} \cup R_i)$.

Lemma 3.2 states that each $Y_\alpha^k \subseteq T_i$ imposes bounds on the length of the interval $[minr_i, maxr_i]$.

Lemma 3.2: $\max r_i + 1 \leq \min r_i - 1 + \min\{\text{step}_\alpha^k | Y_\alpha^k \subseteq T_i\}$.

Proof: By Lemma 3.1(3), $\max r_i + 1 = \min r_i - 1 + \delta(\min r_i - 1, \{n\} \cup R_i)$. Let $Y_\alpha^k \subseteq T_i$. Since $X_\alpha^k \subseteq R_i$, we have $\delta(\min r_i - 1, \{n\} \cup R_i) \leq \delta(\min r_i - 1, \{n\} \cup X_\alpha^k) \leq \text{step}_\alpha^k$. The last inequality follows from Lemma 2.2:

- if $\min r_i - 1 < \min X_\alpha^k$, then, by Lemma 3.1(1), $\min r_i - 1 \geq -1$ and, by Lemma 2.2(2), $\min X_\alpha^k \leq \text{step}_\alpha^k - 1$,
- if $\min r_i - 1 \geq \max X_\alpha^k$, then, by Lemma 3.1(1), $\min r_i - 1 < n$ and, by Lemma 2.2(3), $n - \max X_\alpha^k \leq \text{step}_\alpha^k$.
- otherwise, $\min r_i - 1$ is between two consecutive elements in X_α^k which are at the distance step_α^k , by Lemma 2.2(1).

Let β be the shortest sequence, such that $\min Y_\beta^k = t_1$. If $\beta = ()$, then $t_1 = 0$ and we start binary search from the global root. (Thus each of t_1, \dots, t_e is on distinct level and, hence, $e \leq k + 1$.) Otherwise, let $\beta_0 = \beta$ and, for $j \geq 0$, let β_{j+1} be defined as follows:

- if $\beta_j = ()$, then β_{j+1} is not defined, else
- if $\beta_j = \alpha \cdot (l', l - 1, \dots, l - m)$, where $l + 1 < l'$ and $m \geq 1$, then $\beta_{j+1} = \alpha \cdot (l', l + 1)$, else
- if $\beta_j = (l, l - 1, \dots, l - m)$, where $l < k$ and $m \geq 1$, then $\beta_{j+1} = (l + 1)$, else
- if $\beta_j = (k, k - 1, \dots, k - m)$, where $m \geq 0$, then $\beta_{j+1} = ()$, else
- $\beta_{j+1} = \alpha \cdot (l + 1)$, where $\beta_j = \alpha \cdot (l)$.

Let $\text{last} = \min\{j | \beta_j = ()\}$.

For $0 \leq j \leq \text{last}$, let f_j (the *foot* of β_j) be defined as follows:

- if $\beta_j = \alpha \cdot (l)$, for some α and l , then let $f_j = l$, else
- (i.e. when $\beta_j = ()$) let $f_j = k + 1$.

Note that $f_0 > 0$, since $\min Y_{\alpha \cdot ()}^k = \min Y_\alpha^k$. The following lemma follows directly from the definitions of β_j and last .

- Lemma 3.3:** 1) $f_0 > 0$, and
 2) for $0 \leq j < \text{last}$, $f_j + 1 + |\beta_j| - |\beta_{j+1}| = f_{j+1}$, and
 3) $f_{\text{last}} = k + 1$.

Notice that $\text{last} \leq k$, since $f_0 > 0$, and $f_j < f_{j+1}$ (since $|\beta_j| \geq |\beta_{j+1}|$).

The sequence of time slots $(t_1, (t_1 + 1) \bmod n, \dots, t_e)$ is a prefix of the sequence $\sigma_0 \dots \sigma_{\text{last}}$, where σ_i is the sorted sequence of time slots from $Y_{\beta_i}^k$. Moreover $\sigma_0 \dots \sigma_{\text{last}-1}$ and σ_{last} are increasing sequences of consecutive integers:

- Lemma 3.4:** 1) $\min Y_{\beta_0}^k = t_1$, and
 2) for $0 \leq j < \text{last} - 1$, $\max Y_{\beta_j}^k + 1 = \min Y_{\beta_{j+1}}^k$, and
 3) $\max Y_{\beta_{\text{last}-1}}^k = n - 1$, and
 4) for $0 \leq i \leq \text{last}$, $\emptyset \neq Y_{\beta_j}^k = \{t | \min Y_{\beta_j}^k \leq t \leq \max Y_{\beta_j}^k\}$, and
 5) $Y_{\beta_{\text{last}}}^k = \{0, 1, \dots, n - 1\}$.

We will show the bounds on the sizes of the intersections $U \cap Y_{\beta_j}^k$.

Lemma 3.5: $|U \cap Y_{\beta_0}^k| \leq \lg_2 |Y_{\beta_0}^k| + 1 = \max\{1, f_0\} \leq f_0 + 1$.

Proof: t_1 is the root of the binary search tree $Y_{\beta_0}^k$ and the number of levels of this tree is $\lg_2 |Y_{\beta_0}^k| + 1 = \max\{1, f_0\} \leq f_0 + 1$. ■

Consider the case, when $|\beta_j| = |\beta_{j+1}| \geq 1$.

Lemma 3.6: If $|\beta_j| = |\beta_{j+1}|$ then $|U \cap Y_{\beta_{j+1}}^k| \leq 2 \leq f_{j+1} - f_j + 1$.

Proof: We have $\beta_j = \alpha \cdot (l)$ and $\beta_{j+1} = \alpha \cdot (l + 1)$, for some α and l . If $l = 0$, then $|Y_{\beta_{j+1}}^k| = 1$. Otherwise, let $S = U \cap Y_{\beta_{j+1}}^k$ (time slots used in $Y_{\beta_{j+1}}^k$). If $S = \emptyset$ then $|U \cap Y_{\beta_{j+1}}^k| = 0$. If $S \neq \emptyset$, then let $s = \min\{i | t_i \in S\}$. By Lemma 3.4, we have $Y_{\beta_j}^k \subseteq T_{s-1}$. By Lemma 3.2, $\max r_{s-1} - \min r_{s-1} < \text{step}_{\beta_j}^k = 2 \cdot \text{step}_{\beta_{j+1}}^k$. In $Y_{\beta_{j+1}}^k$ we use only the time slots with the ranks in $[\min r_{s-1}, \max r_{s-1}]$. Hence $|S| \leq |[\min r_{s-1}, \max r_{s-1}] \cap X_{\beta_{j+1}}^k|$. By Lemma 2.4(1), $\min \text{Step}(X_{\beta_{j+1}}^k) \geq \text{step}_{\beta_{j+1}}^k$ and, by Lemma 2.5, $|[\min r_{s-1}, \max r_{s-1}] \cap X_{\beta_{j+1}}^k| \leq 2 = (l + 1) - l + 1 = f_{j+1} - f_j + 1$. ■

Note that if we have ranked κ in the levels $Y_{\alpha \cdot ()}^k, \dots, Y_{\alpha \cdot (l')}^k$, then we have to check at most one rank on each level $Y_{\alpha \cdot (l'')}^k$ with $l'' > l$, since we simply make a continuation of binary search in the binary search tree Y_α^k :

Lemma 3.7: If $\bigcup_{i=0}^l Y_{\alpha \cdot (i)}^k \subseteq T_{e-1}$, then, for each l' such that $l < l' \leq \lg_2 |Y_\alpha^k|$, we have $|U \cap Y_{\alpha \cdot (l')}^k| \leq 1$.

Consider the case, when $|\beta_j| > |\beta_{j+1}| \geq 1$.

Lemma 3.8: If $|\beta_j| > |\beta_{j+1}| \geq 1$ then $|U \cap Y_{\beta_{j+1}}^k| \leq 2 + |\beta_j| - |\beta_{j+1}| \leq f_{j+1} - f_j + 1$.

Proof: Let $m = |\beta_j| - |\beta_{j+1}|$. Let $S = U \cap Y_{\beta_{j+1}}^k$. If $S = \emptyset$ then $|U \cap Y_{\beta_{j+1}}^k| = 0$. If $S \neq \emptyset$, then let $s = \min\{i | t_i \in S\}$. By definition, there is a sequence α and a level number l , such that $\beta_j = \alpha \cdot (l, l - 1, \dots, l - m)$ and $\beta_{j+1} = \alpha \cdot (l + 1)$. We split the binary search tree $Y_{\beta_{j+1}}^k$ into upper part Y' and lower part Y'' as follows: Let $Y' = \bigcup_{i=0}^{l-m} Y_{\alpha \cdot (l+1, i)}^k$ and $Y'' = \bigcup_{i=l-m+1}^l Y_{\alpha \cdot (l+1, i)}^k$. Note that $Y_{\beta_{j+1}}^k = Y' \cup Y''$. By Lemma 3.4, we have $Y_{\beta_j}^k \subseteq T_{s-1}$. Let $X' = \text{revBits}_k(Y')$. In Y' we use only the time slots from $[\min r_{s-1}, \max r_{s-1}]$, thus $|U \cap Y'| \leq |[\min r_{s-1}, \max r_{s-1}] \cap X'|$. By Lemma 2.4(2), $\min \text{Step}(X') \geq 2^{k-(l-m)} = \text{step}_{\beta_j}^k / 2$. By Lemma 3.2, $\max r_{s-1} - \min r_{s-1} < \text{step}_{\beta_j}^k$. Hence, by Lemma 2.5 we have $|[\min r_{s-1}, \max r_{s-1}] \cap X'| \leq 2$. Finally, note that, if $U \cap Y'' \neq \emptyset$, then $Y' \subseteq T_{e-1}$ and, by Lemma 3.7, $|U \cap Y''| \leq m$. And $2 + m = (l + 1) - (l - m) + 1 = f_{j+1} - f_j + 1$. ■

For $j < \text{last}$, let $c_j = |U \cap Y_{\beta_j}^k|$. From Lemmas 3.5, 3.6, and 3.8, we have:

Lemma 3.9: $c_0 \leq f_0 + 1$, and, for $0 < j < \text{last}$, $c_j \leq f_j - f_{j-1} + 1$.

We still need a bound on the number of time slots used since the time slot 0. Let $U' = \{t \in U | t < t_1\}$ (equal to $U \setminus \bigcup_{j=0}^{\text{last}-1} Y_{\beta_j}^k$).

Lemma 3.10: $|U'| \leq k - f_{\text{last}-1} + 2$.

Proof: If $U' = \emptyset$ then the lemma follows. Consider the case $U' \neq \emptyset$: Let $l = f_{\text{last}-1}$. We split the global binary search tree Y_0^k into upper part Y' and lower part Y'' as follows: Let $Y' = \bigcup_{j=0}^l Y_{(j)}^k$ and $Y'' = \bigcup_{j=l+1}^k Y_{(j)}^k$. Let $i' = \max\{i | t_i \geq t_1\}$ (i.e. the index of the last used time slot before the time slot 0). Let $X' = \text{revBits}_k(Y')$. Since the used time slots in U' have ranks in $[\min r_{i'}, \max r_{i'}]$, we have $|U' \cap Y'| \leq |[\min r_{i'}, \max r_{i'}] \cap X'|$. Since $Y_{\beta_{\text{last}-1}}^k \subseteq T_{i'}$, we have, by Lemma 3.2, $\max r_{i'} - \min r_{i'} < \text{step}_{\beta_{\text{last}-1}}^k$. Since,

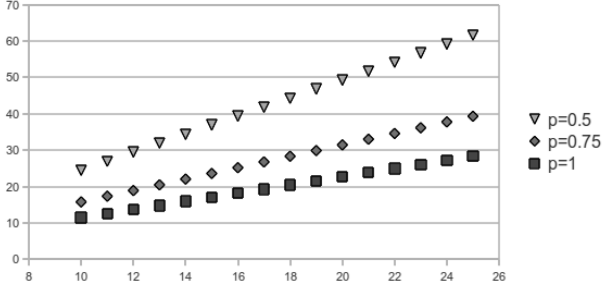


Fig. 2. Average energy used, for $10 \leq k \leq 25$, for probabilities of successful reception $p \in \{0.5, 0.75, 1\}$.

by Lemma 2.4(2), $\min\text{Step}(X') \geq 2^{k-l} = \text{step}_{\beta_{last-1}}^k / 2$, we have, by Lemma 2.5, $|\llbracket \min r_{i'}, \max r_{i'} \rrbracket \cap X'| \leq 2$. Finally, note that, if $U' \cap Y'' \neq \emptyset$, then $Y' \subseteq T_{e-1}$ and, by Lemma 3.7, $|U' \cap Y''| \leq k - l$. ■

Now, we can bound $|U|$:

Lemma 3.11: $|U| \leq 2 \cdot k + 2$.

Proof: We have $|U| = \sum_{j=0}^{last-1} c_j + |U'|$. By Lemma 3.9, we have $\sum_{j=0}^{last-1} c_j \leq f_0 + 1 + \sum_{j=1}^{last-1} (f_j - f_{j-1} + 1) = last + f_{last-1}$. By Lemma 3.10, we have: $|U'| \leq k - f_{last-1} + 2$. Since $last \leq k$, we have $(last + f_{last-1}) + (k - f_{last-1} + 2) \leq 2k + 2$. ■

Lemma 3.11 completes the proof of Theorem 3.1. ■

Remark. Note that the bound is quite precise: Consider the case when $\kappa_{n/2} < \kappa < \kappa_{n/2+1}$ and $t_1 = \min Y_{(2)}^k$. Then, on each level $Y_{(2)}^k, \dots, Y_{(k)}^k$, we are using two slots and (in the next round) we are using one slot in $Y_{(1)}^k$. Thus the total number of the used slots is $2(k-1) + 1 = 2k - 1$.

Theorem 3.1 states the bound on energy under the assumption that every message is received with probability $p = 1$. On Figure 2 we present results of simulations of the basic algorithm under the assumption that the probability of successful reception is p . (If the reception is unsuccessful, then a unit of energy is used in the corresponding time slot, however the interval $[\min r, \max r]$ is not updated.) The horizontal axis is k , where 2^k is the length of the broadcast sequence, and the vertical axis is the average energy used by the receiver in 100000 tests. In each test, a random starting time slot t_0 , $0 \leq t_0 < 2^k$, and a key (not present in the broadcast sequence) with random rank between 0 and 2^k have been uniformly selected. Since the key is not present in the sequence, the expected time is bounded by $(1/p^2 - 1/2) \cdot 2^k$.

IV. COMPUTATION OF nextSlotIn

The function $\text{nextSlotIn}_k(t, r_1, r_2)$ is recomputed by RBO whenever it has to find the next time slot after t , such that the rank of the key transmitted in this slot is contained in the interval $[r_1, r_2]$. If the rank of the searched key is between r_1 and r_2 , then RBO can skip all the messages transmitted between time slots $t + 1$ and $\text{nextSlotIn}_k(t, r_1, r_2) - 1$. Efficient computation of this function reduces the time and the energy used by the processor of the receiver device. If $2^k / (r_2 - r_1)$ is not too large (e.g. below one hundred) then the

distance between consecutive elements of $\text{revBits}([r_1, r_2])$ is not large and we may *naively* check sequentially the ranks of the time slots $(t+1) \bmod 2^k, (t+2) \bmod 2^k, \dots$. Otherwise, if $r_2 - r_1$ is a small number, then we may apply *reverse* searching among time slots $\text{revBits}(r_1), \dots, \text{revBits}(r_2)$, for the nearest successor of t . We propose polylogarithmic time computation of nextSlotIn , that should be applied when both $2^k / (r_2 - r_1)$ and $r_2 - r_1$ are large. The implementation of this algorithm in programming language can be found in [13]. Here we describe its idea and a more intuitive pseudo-code. First, let us see how to compute the (globally) minimal time slot t , such that $\text{revBits}_k(t) \in [r_1, r_2]$. Let $\min\text{RevBits}_k(r_1, r_2) = \min \text{revBits}_k(\{x \mid r_1 \leq x \leq r_2\})$. Note that if x is (the rank of) the node of the binary search tree, then the left (respectively, right) child of x is $x_L = x - 2^{k-l-1}$ (respectively, $x_R = x + 2^{k-l-1}$), where l is the level of x . We can compute $\min\text{RevBits}_k$ by following the the path in the binary search tree until we enter the interval $[r_1, r_2]$ (see Algorithm 2). By symmetry of revBits_k , we have that

```

function  $\min\text{RevBits}_k(r_1, r_2)$ 
 $x \leftarrow 0; s \leftarrow 2^{k-1};$ 
while  $x < r_1$  or  $x > r_2$  do
  if  $x < r_1$  then  $x \leftarrow x + s$  else  $x \leftarrow x - s$ 
   $s \leftarrow s/2;$ 
return  $\text{revBits}_k(x);$ 

```

Algorithm 2: Computing $\min\text{RevBits}$

$\max\text{RevBits}_k(r_1, r_2) = \max \text{revBits}_k(\{x \mid r_1 \leq x \leq r_2\})$ is equal to $2^k - \min\text{RevBits}_k(2^k - r_2, 2^k - r_1)$.

Here is the outline of our algorithm for computing $\text{nextSlotIn}_k(t, r_1, r_2)$:

- 1) If $\text{revBits}_k(t)$ is *only* one side of the interval $[r_1, r_2]$, then remove it:
 - If $r_1 < r_2$ then:
 - if $\text{revBits}_k(t) = r_1$, then $r_1 \leftarrow r_1 + 1$,
 - else if $\text{revBits}_k(t) = r_2$, then $r_2 \leftarrow r_2 - 1$.
- 2) If $[r_1, r_2]$ is a singleton then there is no choice:
 - If $r_1 = r_2$ then **return** $\text{revBits}_k(r_1)$.
- 3) If t is still before the first slot ranked in $[r_1, r_2]$ in this round, the return the first slot ranked in $[r_1, r_2]$:
 - Let $t\text{First} = \min\text{RevBits}_k(r_1, r_2)$.
 - If $t < t\text{First}$ then **return** $t\text{First}$.
- 4) If $t + 1$ is after the last slot ranked in $[r_1, r_2]$, then return the first slot ranked in $[r_1, r_2]$ in the next round of broadcasting:
 - Let $t\text{Last} = \max\text{RevBits}_k(r_1, r_2)$.
 - If $t\text{Last} \leq t$ then **return** $t\text{First}$.
- 5) Here, $t\text{First} \leq t < t\text{Last}$.
 - Find minimal level l , such that $l \geq \lceil \lg_2(t + 1) \rceil$ and $\min L = \min\{i \mid 2^{k-l} + i \cdot 2^{k-l+1} \geq r_1\}$ is not greater than $\max L = \max\{i \mid 2^{k-l} + i \cdot 2^{k-l+1} \leq r_2\}$.

Such l is the first level (starting from the level of t) that intersects $[r_1, r_2]$ and $\{\min L, \dots, \max L\}$ are the

coordinates within the level l of this intersection. Note that $\min L = \lceil (r_1 - 2^{k-l})/2^{k-l+1} \rceil = \lfloor (r_2 + 2^{k-l} - 1)/2^{k-l+1} \rfloor$, and $\max L = \lfloor (r_2 - 2^{k-l})/2^{k-l+1} \rfloor$. The number of nodes above the level l (and also the size of the level l) is 2^{l-1} .

- Let $\text{above}L = 2^{l-1}$.
 - Let $t\text{First}L = \min\text{RevBits}_{l-1}(\min L, \max L)$ (the first time slot of the level l ranked within the level l in $[\min L, \max L]$).
- 6) $\text{above}L + t\text{First}L$ is the global number of the first time slot of the level l ranked within the level l in $[\min L, \max L]$. Check whether t is still before this time slot:
- If $t < \text{above}L + t\text{First}L$ then **return** $\text{above}L + t\text{First}L$.
- 7) Here l is the level of t , since we did not return in previous step.
- Let $t\text{Last}L = \max\text{RevBits}_{l-1}(\min L, \max L)$.
- 8) If $t \geq \text{above}L + t\text{Last}L$ then (we have to find the first slot in $[r_1, r_2]$ below the level l):
- a) Find minimal level $l_1 > l$, such that $\min L_1 = \min\{i \mid 2^{k-l_1} + i \cdot 2^{k-l_1+1} \geq r_1\}$ is not greater than $\max L_1 = \max\{i \mid 2^{k-l_1} + i \cdot 2^{k-l_1+1} \leq r_2\}$. (l_1 is the next level after l that intersects $[r_1, r_2]$.)
 - b) Let $\text{above}L_1 = 2^{l_1-1}$ (the number of nodes above the level l_1).
 - c) Let $t\text{First}L_1 = \min\text{RevBits}_{l_1-1}(\min L_1, \max L_1)$.
 - d) **Return** $\text{above}L_1 + t\text{First}L_1$.
- 9) Here $t\text{First}L \leq t - \text{above}L < t\text{Last}L$ and we search within the level l (tail recursion):
- **Return** $\text{above}L + \text{nextSlotIn}_{l-1}(t - \text{above}L, \min L, \max L)$.

The depth of the recursion is at most k , since each level has no more than a half of the nodes of the tree. Step 8a is performed only on the last recursion. In step 5, t is above level l only on the last recursion. Thus, the algorithm performs $O(k)$ elementary operations such as revBits , $\min\text{RevBits}$, $\max\text{RevBits}$ or arithmetic operations. Since each such operation needs $O(k^2)$ bit operations, the total cost is $O(\log^3 n)$ of bitwise operations. We replace *tail recursion* by iterative version (see the code of `plogNextSlotIn` at [13]). Thus RBO uses only constant number of $\lceil \log_2 n \rceil$ -bit variables.

V. IMPLEMENTATION OF THE PROTOCOL

We propose an outline of practical implementation of RBO for realistic model, where the clocks of the sender and of the receiver are not perfectly synchronized. We also have to take into account the possible delays in processing the received messages by the underlying system protocols. We have arbitrarily selected the set of available RBO services. In the case of tiny devices such as sensors, it is customary that the code of the protocol implementation is modified and tailored to the particular needs of the (single) application run on the device.

A. RBO Message Format

The RBO message consists of a header and an arbitrary payload. The header contains the following fields:

- *sequenceId*: The identifier of the sequence. If sequence of keys changes it should be changed. Zero is reserved for invalid identifier - should not be used.
- *logSequenceLength*: Logarithm to the base of 2 of the sequence length. The length of the sequence is integer power of two.
- *timeSlotLength*: Time interval between the starts of consecutive message transmissions (e.g. in milliseconds).
- *key*: The key of the message.
- *rank*: The rank of the *key* in the transmitted sequence. Thus the time slot of this message is $\text{revBits}_{\log\text{SequenceLength}}(\text{rank})$.

B. Sender's Part of the RBO

If the length n of the sequence to be transmitted is not an integer power of two, then some of the messages should be doubled to extend the length to the power of two $n' = 2^{\lceil \lg_2 n \rceil}$. Note that the distance between consecutive occurrences of the doubled keys in periodic broadcasting reduces to $2^{\lceil \lg_2 n \rceil - 1}$, while the distance between occurrences of the not doubled keys increases to $2^{\lceil \lg_2 n \rceil}$. To compensate for this "injustice", we can increase the length of the sequence to even higher power of two by creating more balanced numbers of copies of the messages.

The sender broadcasts in rounds the sequence of messages sorted by the keys and permuted by the revBits permutation. The messages should have properly filled in header fields. Whenever the sequence of keys changes, the field *sequenceId* must be changed unless *logSequenceLength* is changed.

C. Receiver's Part of the RBO

The RBO module on the receiver's device offers to its user application a *split-phase* interface. Such interface (see [14]) consists of the *commands* to be called by the user and *events* to be signalled to the user by the protocol. The user (i.e. the running application) issues a command *search(key)* that initiates the search and returns immediately. As soon as the search is finished, the event call-back *searchDone(message, error)* is posted to be signaled to the user, where *message* is the buffer containing the searched message (if found), and *error* is the status of the search result:

- **SUCCESS** (the message has been found),
- **KEY_NOT_PRESENT** (the *key* is not in the sequence),
- **TIMEOUT** (no RBO messages has been received for long time),
- **BAD_MESSAGE** (an RBO message with *sequenceId* = 0 has been received),
- **FAILED_RADIO** (problems detected when switching the radio on/off).

The user can also pause the current search with the command *stop()* (to be resumed later) or abandon it with the command *reset()* (forgetting all partial results of the search).



Fig. 3. State diagram of the RBO receiver protocol.

On the other hand RBO uses the system modules and interfaces that provide the timers (*timeoutTimer*, *sleepingTimer*), and the means (e.g. delivered by the TinyOS module *ActiveMessageC*) of packet reception (e.g. the interface *Receive*) and of switching the radio on and off (e.g. the interface *SplitControl*). RBO can be in one of the three states:

- IDLE (when RBO is not used),
- LISTENING (when radio is switched on),
- SLEEPING (when radio is switched off until *sleepingTimer* fires).

The possible state transitions are displayed on Figure 3. In transition to LISTENING, a *sleepingTimer* is canceled, *timeoutTimer* is set and the radio is switched on. (Actually, a split-phase process of switching the radio on is initiated.) In transition to SLEEPING, the *timeoutTimer* is canceled, *sleepingTimer* is set and radio is switched off. In transition to IDLE, the timers are canceled.

RBO has following variables:

- *searchedKey* – the recently searched key,
- *logSequenceLength* and *sequenceId* (initiated to zero) – recently received in RBO message,
- *minRank* and *maxRank* – learned lower and upper bound on the rank of *searchedKey*.

The user's command *search(key)* compares *key* to *searchedKey* and initiates searching:

- If *key* < *searchKey*, then set *minRank* to zero.
- If *key* > *searchKey*, then set *maxRank* to $2^{\logSequenceLength} - 1$.
- Set *searchedKey* to *key* and switches RBO to LISTENING state. (Thus we may take advantage from the most recent search.)

The *stop* and *reset* commands switch RBO to IDLE. (Moreover, *reset* sets *sequenceId* to zero.)

RBO implements callbacks of the events signalled by the timers and the interfaces *Receive* and *SplitControl*. The *timeoutTimer* event *fired()* in the state LISTENING causes RBO transition to the state IDLE and signalling *searchDone(..., TIMEOUT)* to the user. The *sleepingTimer* event *fired* in the state SLEEPING causes RBO transition to the state LISTENING and switching the radio on. The (most essential) event *received(message)* (reception of the *message*) signalled by the radio *Receive* interface to RBO in state LISTENING is served by RBO as follows (we use notation *message.name* to denote the field in the *message* header and *name* to denote variable of RBO):

- 1) *timeoutTimer* is canceled.
- 2) If *message.sequenceId* = 0, then RBO switches to IDLE and signals *searchDone(message, BAD_MESSAGE)*, and returns.
- 3) If *message.sequenceId* \neq *sequenceId* or *message.logSequenceLength* \neq *logSequenceLength*, then

- set *sequenceId* to *message.sequenceId*,
- set *logSequenceLength* to *message.logSequenceLength* and
- (forget old bounds) set *minRank* to 0 and *maxRank* to $2^k - 1$, where $k = \logSequenceLength$.

- 4) If *message.key* = *searchedKey* then RBO switches to IDLE and signals *searchDone(message, SUCCESS)*, and returns.
- 5) Try to update the bounds on the rank:
 - If *message.key* > *searchedKey* and *message.rank* \leq *maxRank* then set *maxRank* to *message.rank* + 1, else
 - if *message.key* < *searchedKey* and *message.rank* \geq *minRank* then set *minRank* to *message.rank* - 1.
- 6) Test for absence of the *searchedKey*:
 - If *minRank* > *maxRank*, then RBO switches to IDLE and signals *searchDone(message, KEY_NOT_PRESENT)*, and returns.
- 7) Compute the time remaining to the next useful message:
 - Let $k = \logSequenceLength$ and $now = revBits_k(message.rank)$ and $next = nextSlotIn_k(now, minRank, maxRank)$.
 - If $now < next$ then let $slotsToNext = next - now$, else let $slotsToNext = 2^k - now + next$.
 - Let $remainingTime = slotsToNext \cdot message.timeSlotLength$.
- 8) If *remainingTime* is greater than a threshold (i.e. *minSleepingTime*), then RBO sets *sleepingTimer* to *remainingTime* - *relativeMargin* - *timeMargin*, switches the radio off and transits to state SLEEPING, where *timeMargin* is some constant margin (e.g. few milliseconds) that should compensate for radio switching on and off delays and the delay in message processing, and *relativeTimeMargin* = *remainingTime*/*d* should compensate for not ideal synchronization of the sender's and receiver's clocks. (If *d* is a power of two, then the division may be replaced by a binary shift.)
- 9) Otherwise (i.e. when *remainingTime* < *minSleepingTime*), only the *timeoutTimer* is restarted.
- 10) RBO returns.

We skip the descriptions of the implementations of the callbacks *startDone* and *stopDone* of the interface *SplitControl* used for switching the radio on and off. In practice RBO may receive some overhead messages due to the hardware delays and to keep synchronization with the sender. The proper balancing of the parameters (such as *minSleepingTime*, and the absolute and the relative time margins) that control the tradeoff between the energy savings and the reliability can be subject of real life experiments.

VI. CONCLUSION

This paper proposes an efficient solution to the problem of transmitting very long streams of uniform messages for selective reception by battery powered receivers.

We proposed an implementation of the protocol based on a very simple basic algorithm (Algorithm 1) and an efficient algorithm for computation of its essential function *nextSlotIn*. Thus, the protocol can be implemented on devices with very weak processors and with very limited memory.

Note, that we can “plug-in” arbitrary permutation instead of bit-reversal in the basic algorithm. We have shown that, for the bit-reversal permutation, the number of necessary receptions is bounded by $2\lceil\log_2 n\rceil + 2$. On the other hand we have shown an example, where $2\lceil\log_2 n\rceil - 1$ receptions are necessary. It is interesting question, whether there exist any permutation, for which the respective bounds are lower than for bit-reversal. However, $\log_2 n$ is an obvious lower bound and the simplicity of bit-reversal is a great advantage in possible implementations. The tests for unreliable transmissions (Figure 2) show that the expected energetic costs are very low even if the probability of successful reception is much lower than one.

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